COMPOSITE THEORY OF LARGE ANGLE SCATTERING AND NEW TESTS OF PARTON CONCEPTS*

J. F. GUNION, S. J. BRODSKY and R. BLANKENBECLER
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305, *USA*

Received 25 April 1972

A formalism for handling the scattering of composite systems is presented. Using this formalism, the contribution to bound state scattering at large energy and angle from the interchange force is calculated. This force leads to elastic cross sections which behave at fixed angle like the sixth power of the form factor and for which $(d\sigma/dt)/(d\sigma/dt)_{90^0}$ is an energy independent function of z. Comparison with p-p and π-p data shows excellent agreement. A new test of the existence of a pointlike coupling of the photon to partons is suggested.

If hadrons are composite, as indicated by the recent SLAC deep inelastic electron nucleon scattering measurements, then they will interact simply by the interchange of their constituents – just as electron interchange provides a force in elastic atom-atom scattering. In the special case of the quark model, two protons can scatter through the interchange of a single n or p quark. In fact we shall argue that in the region of large energy and momentum transfer, where coherent regge effects may be small, such interchange interactions are the dominant hadronic force. Thus the large s, t and u dependence of exclusive scattering processes is determined by the bound state wave function at small distances. In our calculations we utilize the fact that the form factors determine this short distance behavior of the bound state wave function as well as its dominant spin structure. Drell and Yan [1] have shown that the threshold behavior of the deep inelastic structure function $F_2(\omega)$ is determined by this same region of the wave function.

The interchange force leads to an asymptotic fixed angle cross section which contains six factors of the elastic form factor, rather than four as in the Wu-Yang model [2]. In addition the interchange theory predicts that the ratio of the differential cross section to its value at 90 degrees is a universal function of $z = \cos\theta_{CM}$ for large t and u. This model also predicts quite different angular distributions for processes which should look the same in the Wu-Yang model, such as K^+p and K^-p, and that there should be a striking difference in the energy dependence of the 90^0 cross sections of photon processes relative to their corresponding vector meson analogues.

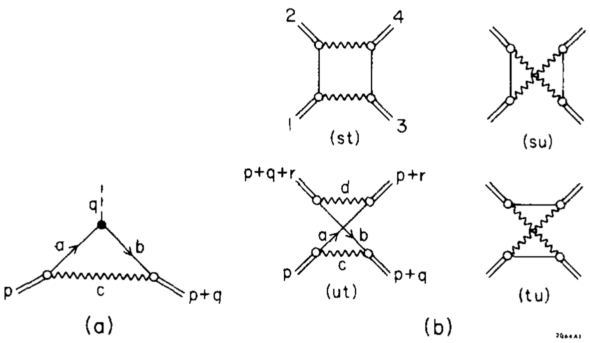

Fig. 1. a) Form factor diagram. b) Basic scattering topologies.

In this paper we introduce a natural formalism for covariant bound state computations utilizing old fashioned perturbation theory ** in the infinite momentum frame. We illustrate the technique and its advantages by considering the G_E and G_M form factor of the nucleon in a composite model ††. The diagram for this vertex is shown in fig. 1a. For the case of all spinless particles, each charged parton constituent a, contributes to the elastic form factor a term of the form

* Work supported by the US Atomic Energy Commission.

** The utility of old fashioned perturbation theory in the infinite momentum frame was first made apparent in the calculations of Drell et al.[3].

†† For parallel computational techniques for electromagnetic processes using the Bethe Salpeter equation see Drell and Lee [4].

$$F_a(q^2) = \lambda_a \int d^2k \int_0^1 \frac{dx}{x(1-x)} \psi_1(\boldsymbol{k}_\perp)\psi_2(\boldsymbol{k}_\perp+(1-x)\boldsymbol{q}_\perp) \quad (1)$$

where

$$p_\mu = (P+M^2/2P,\ \boldsymbol{0}_\perp, P)$$
$$q_\mu = (q\cdot p/2P, \boldsymbol{q}_\perp, 0),\quad q^2 = -\boldsymbol{q}_\perp^2 = -2q\cdot p, \quad (2)$$

and the theory is defined in a limiting frame where $P \to$ infinity. The onshell intermediate particle momenta are parameterized as

$$\boldsymbol{p}_a = \boldsymbol{k}_\perp + x\boldsymbol{p},\quad \boldsymbol{p}_b = \boldsymbol{k}_\perp + \boldsymbol{q} + x\boldsymbol{p}$$
$$\boldsymbol{p}_c = -\boldsymbol{k}_\perp + (1-x)\boldsymbol{p}. \quad (3)$$

The bound state wave function $\psi_1(\boldsymbol{k}_\perp)$ in eq. (1) describes the breakup of the composite system into an elementary parton constituent and the remaining core † which we treat as a single particle. The core represents the combined effect of the remaining constituents. Generally we may write ψ_1 as

$$\psi_1(\boldsymbol{k}_\perp) = (M_1^2 - S_{ac} + i\epsilon)^{-1}\phi(S_{ac}) \quad (4)$$

where the two particle invariant mass S_{ac} is given by

$$S_{ac} = (p_a+p_c)^2 = \frac{\boldsymbol{k}_\perp^2 + m_a^2(1-x) + x m_c^2}{x(1-x)}. \quad (5)$$

The vertex function, ϕ, reduces to a coupling constant in the case of a lowest order Feynman diagram calculation.

If $\psi \sim S^{-n}$ (6)

then

$$F(q^2) \sim \int_0^{1-O(m^2/q_\perp^2)} \frac{dx\, x^{2n-1}}{1-x} N_\psi(x) \frac{1}{q_\perp^{2n}}$$
$$\sim (\boldsymbol{q}_\perp^2)^{-n} \log \boldsymbol{q}_\perp^2. \quad (7)$$

The function $N_\psi(x)$ is given by

$$N_\psi(x) = \int d^2k \frac{\psi(\boldsymbol{k}_\perp)}{x^n(1-x)^n}. \quad (8)$$

The fact that ψ depends only on the covariant variable S guarantees that N_ψ is a smooth function of x and that the Drell Yan threshold relation, $F_2^p(x) = (1-x)^{2n-1}$ for $x \to 1$, is satisfied ††

The inclusion of spin is straightforward. If we assume that the proton is a bound state of a spin 1/2 parton and a spin 1 core (with γ_μ coupling), then we obtain the scaling law: $G_M \sim G_E$ for large t †††. The dipole behavior of the form factor results if $\psi(S) = S^{-3}$. If the core has both spin 1 and spin 0 components (as is natural in the quark model) with the same wave function behavior, then the asymptotic scaling relation is unchanged.

We turn now to a consideration of the scattering of two bound states. The interchange mechanism leads, in general, to three different graph topologies. These are shown in fig. 1b which also shows that the *ut* topology must, in general, be symmetrized with respect to the role of the partons and cores. This is also true of the *st* and *su* topologies. In the case of pp scattering only the last two diagrams contribute when one uses the valence quark model assignments for the constituents, i.e., the parton can only be a p or an n quark and the remaining core must have the quantum numbers of pn or pp respectively. For simplicity, we will continue to use the quark model as a guide, but our results are true in many other models as well.

As an example we give the result for the *tu* diagram using the labelling of fig. 1b. Summing over the four allowed time orderings, we obtain

$$M(u,t) = \int d^2k_\perp \int_0^1 \frac{dx}{x^2(1-x)^2} \Delta\ \psi_1(\boldsymbol{k}_\perp) \times$$
$$\psi_3(\boldsymbol{k}_\perp+(1-x)\boldsymbol{q}_\perp)\,\psi_2(\boldsymbol{k}_\perp+(1-x)\boldsymbol{q}_\perp - x\boldsymbol{r}_\perp)\psi_4(\boldsymbol{k}_\perp - x\boldsymbol{r}_\perp) \quad (9)$$

where the transverse vectors $\boldsymbol{r}_\perp$ and $\boldsymbol{q}_\perp$ satisfy $\boldsymbol{r}_\perp\cdot\boldsymbol{q}_\perp = 0$, $u = -\boldsymbol{r}_\perp^2$, and $t = -\boldsymbol{q}_\perp^2$‡. Sur-

† The possible consistency of a model in which the partons interact as though they were free in certain kinematic regions but cannot exist as free states has been investigated by Johnson [5].

†† In general ϕ may also depend on the invariant $p\cdot(p_a - p_c)$ corresponding to the relative energy in the center of mass system. However, we shall assume that the asymptotic behavior of the wave function is given entirely in terms of S or $\boldsymbol{p}^2$cm, as in the case of potential or ladder approximation models. A dominant dependence upon the variable $p\cdot(p_a - p_c)$ would cause the 90° cross sections to fall far too slowly and is inconsistent with the Drell-Yan threshold relation. Note that both variables depend only on the component of particle a's momentum transverse to the momentum of the bound state. This is a general consequence of rotational invariance.

††† We disregard logarithmic modifications throughout the paper.

‡ The vectors $\boldsymbol{r}$ and $\boldsymbol{p}_d$ are chosen to be

$$\boldsymbol{r} = (r_\perp^2/2p,\ \boldsymbol{r}_\perp,\ 0)$$
$$\boldsymbol{P}_d = \left((1-x)P + \frac{m_d^2 + (\boldsymbol{r}_\perp - \boldsymbol{k}_\perp)^2}{2(1-x)P},\ \boldsymbol{r}_\perp - \boldsymbol{k}_\perp,\ (1-x)P\right).$$

prisingly, the familiar energy denominator, $\Delta = E_1 + E_2 - E_a - E_b - E_c - E_d$, appears in the numerator. It is given by

$$\Delta = M_1^2 + M_2^2 - \frac{(k_\perp - x r_\perp)^2 + (k_\perp + (1-x) q_\perp)^2 + m_x}{x(1-x)} \tag{10}$$
$$2m_x^2 \equiv x(m_c^2 + m_d^2) + (1-x)(m_a^2 + m_b^2) .$$

For proton-proton scattering the asymptotic behavior is

$$M(u,t) \approx -4 \int_0^1 dx\, N_\psi(x) x^{n-2} (1-x)^{n-2} \times \tag{11}$$
$$\frac{x^2 r_\perp^2 + (1-x)^2 q_\perp^2}{x(1-x)} \psi((1-x) q_\perp) \psi(-x\, r_\perp) \psi((1-x) q_\perp - x\, r_\perp) .$$

With the inclusion of spin which leads to additional spin or kinematic factors under the integral sign ††, the final asymptotic result for p-p scattering is †††

$$\left(\frac{d\sigma}{dt}\right)_{pp} \propto \frac{R(z)}{s^{12}} \tag{12}$$

where $R(z) = f(z)(1 - z^2)^{-6}$, $z = \cos\theta_{CM}$. $f(z)$ is a slowly-varying function of z for z^2 not near 1.

This formula is compared with the data [6] in figs. 2A and 2B. The differential cross section at 90° is seen to be consistent with s^{-12} for $s \geq 10 (\mathrm{GeV})^2$. The highest energy points may favor a slope of 14, which would result if the form factor behavior were $t^{-2.3}$ instead of t^{-2} near $t = 20$. Interestingly enough, the form factor does appear to fall faster than a dipole in this region. The fact that $R(z)$ is independent of energy is well-verified by the data plotted in fig. 2B. The angular dependence of this curve is very close to that of the predicted $(1 - z^2)^{-6}$ for $|t|$ and $|u| > 4\ (\mathrm{GeV})^2$.

Similar results can be obtained for all two-body hadron-hadron and photon-hadron scattering processes. For the case of π-p scattering,

†† Z graphs in which there is a backward moving particle are eliminated in bound state theory even in the presence of spin because of the extra fall-off of ϕ. This is to be contrasted with the situation in two photon processes [9].

††† For the spinless-case eq. (11) gives the same s-dependence with the z dependence given by $f(z) \times (1 - z^2)^{-4}$. Thus spin effects can alter the angular dependence.
We have also examined the effects of absorption on the basic interchange interaction. Since absorption has a much larger range than the interchange force, its main effect is to change the normalization without affecting the energy or angular prediction.

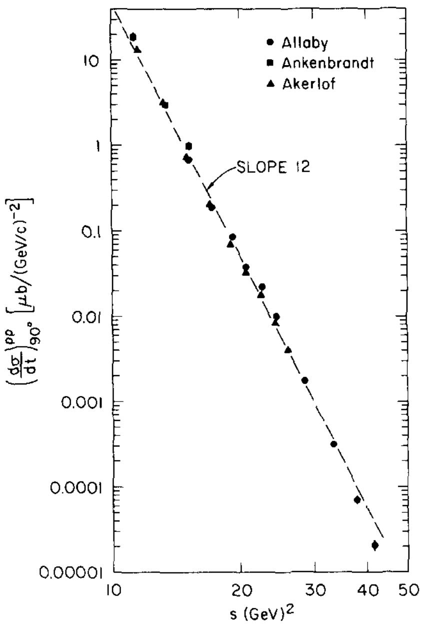

Fig. 2A. Log-log plot of $d\sigma/dt$ at 90° versus s for p-p scattering.

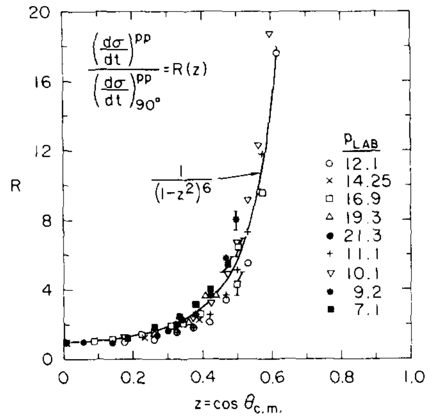

Fig. 2B. A plot of $R(z)$ at various energies for p-p scattering. Sample errors are shown.

where the pion is taken as a bound state of a spin 1/2 parton and a spin 1/2 core, both the and ut diagrams of fig. 1b contribute. If we assume that the asymptotic power dependence of the pion form factor is t^{-l}, then the result for $\pi^\pm$p is

$$\left(\frac{d\sigma}{dt}\right)_{\pi^\pm p} \propto |H^\pm(z)|^2 / s^{4+4l} = R^\pm(z)/s^{4+4l} , \tag{13}$$

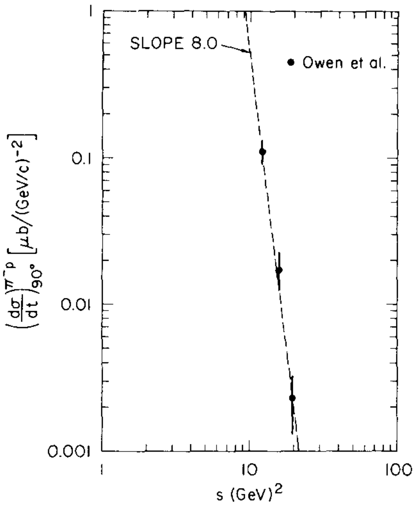

Fig. 3A. Log-log plot of $d\sigma/dt$ at 90° versus s for π^-p scattering.

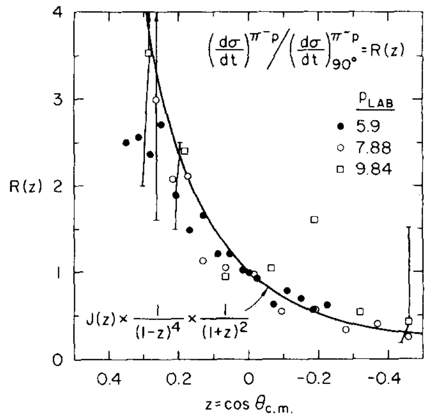

Fig. 3B. A plot of $R^-(z)$ at the three highest available energies for π^-p scattering. $J(z)$ is predicted from eq. (13) to be $J(z) = \frac{1}{9}[1+2(1+z)^{3/2}]^2$. Sample errors are shown.

with $H^+(z) = 2H^{ut} + H^{st}$, $H^-(z) = H^{ut} + 2H^{st}$, and $H^{ut} = g^{ut}(z)(1-z)^{-2}(1+z)^{-1}$, $H^{st} = g^{st}(z)(1-z)^{-2}(1+z)^{1/2}$.

The g's are slowly varying functions of z for z^2 not near 1. H^{st}, which in the present case has only a small imaginary part, is similar in magnitude to H^{ut} for all $z \geq 0$. This, of course, tells us that the π^+p cross section should be roughly equal to the π^-p cross section in this region. The ut terms, however, dominate for sufficiently negative z and lead to a rising cross section there.

In fig. 3A we have plotted the 90° π^-p cross section [7] and conclude that it falls approximately as s^{-8}, requiring $l = 1(\psi_\pi \sim S^{-3/2}$ for spin 1/2 constituents). In fig. 3B we have plotted $R^-(z)$ as calculated from the experimental data available at three different energies. Again it is certainly consistent with an energy independent universal function of z which is very much like that predicted using eq. (13). The asymmetrical behavior of $R(z)$ around $z = 0$ is due primarily to the different asymptotic behavior of the pion and nucleon wave functions, which breaks the symmetry of the ut term, and to the intrinsic asymmetry of the st term. The overlain curve is calculated by setting $g^{st} = g^{ut}$.

A similar analysis can be made for $K^\pm p$ scattering. Using the quark model assignments the exotic reaction K^+p has only a ut term while K^-p has only an st contribution. If the K form factor falls at the same rate as the π^- form factor, then the energy dependence is the same as for the πp case. K^+p should be relatively flat when compared to the K^-p cross section; the latter should have no enhancement for z near -1, (until one reaches the exotic backward peak), while having the same forward peaking as the former. The 5 GeV data [8] indicates that this is indeed the case, but higher energy data is required before a meaningful comparison can be made in a region without regge effects. (In some sense the mere presence of a nonexotic backward peak in K^+p demands that the above contrast be present.) In addition, the interchange theory predicts that the K^+p amplitude is real while the K^-p amplitude is almost purely real, and that the K^+p cross section should be somewhat larger than K^-p at 90° *.

In the parton model, the photon couples in a point-like fashion to the hadronic constituents. Thus proton processes will fall more slowly in energy than their corresponding vector meson analogues when the interchange mechanism dominates. For example the differential cross section for photo pion production at 90° is predicted to fall as $s^{-13/2}$ (for spin 1/2 partons), whereas the cross section for $\pi p \to \rho p$ at $t = u$ is predicted to fall at least as fast as s^{-8}, if the ρ form factors fall at least as fast as the pion form factor. Comparison of these reactions at large energy and angle is a crucial test of the validity of this

* It is interesting that the 5 GeV/c data suggests that the former process has more striking interference minima than the latter. This is consistent with the interfering K^+p Regge contributions being purely real and K^-p Regge terms being purely rotating phase as suggested by the usual exchange degenerate picture. A thorough analysis should be worthwhile.

model and the existence of a point-like coupling of the photon**.

So far we have made little use of the selection rules imposed by the valence quark model assignments for the hadronic parton and core constituents. Some additional predictions are for instance that pp and np elastic scattering cross sections should become identical at large energy and angle. In particular n-p should become symmetric about 90^o. In addition both receive contributions only from *ut* diagrams which are purely real; hence neither should exhibit any polarization in this region. In comparison $\bar{p}$ - p which receives contributions purely from the *st* topology should exhibit some polarization and be strongly skewed in favor of $z \geqslant 0$, although for fixed z it should fall in s at the same rate as p-p(s^{-12}).

In general the meson nucleon interactions receive contributions from both *st* and *ut* topologies but always in a well defined ratio. The angular dependence away from 90^o can be different from process to process (though some are identical; e.g., $\pi^o p \rightarrow \pi^o p$: $\eta p \rightarrow \eta p$: $\eta p \rightarrow \pi^+ n = 3:1:\sqrt{2/3}$ in the amplitudes) but all fall with the same s^{-8} power. For "charge exchange" reactions such as $\pi^- p \rightarrow \pi^o n$ the *st* and *ut* terms enter with opposite signs, unlike the elastic scattering cases, so that one could expect the charge exchange reactions to be up to an order of magnitude smaller at 90^o. In general those meson nucleon interactions which receive contributions from *st* topologies could be expected to exhibit some polarization, although estimates tend to indicate that the imaginary part of the *st* contribution is always small compared to its real part. Certain of the kaon reactions, e.g., K^+-p, receive no *st* contributions and should not exhibit polarization.

We also note that if the γ_μ coupling of the spin 1 core of the nucleons is present, then asymptotically one should see helicity conservation in the meson nucleon interactions, total helicity conservation in p-p scattering, but helicity should not be conserved in γ-p $\rightarrow$ π-p at large angles. In the case of ρ-p $\rightarrow$ ρ-p, helicity should again be conserved.

Finally, measurements of the process p-p $\rightarrow$ nΔ^{++} can be used to determine the Δ wave function and to check the zero polarization prediction for such a process at large s, t and u. In general, any quasi-two-body reaction will yield new information concerning the wave function of the produced particle and its constituents.

In conclusion, we have presented a particularly simple explanation of large s, large angle scattering which agrees quite well with the existing data. It provides, via systematic comparison of relative normalizations, energy and angle dependencies, a unique opportunity to determine the wave functions of the hadrons and the properties, e.g., quantum numbers and spin, of their constituents.

We wish to acknowledge helpful conversations with S. Berman, J. Bjorken, M. Davier and R. Roskies.

** The consequences of this point-like coupling for two photon processes have been discussed by Brodsky et al. [9]. These papers provide further examples of some of the calculational techniques used here.

References

[1] S. D. Drell and T. M. Yan, Phys. Rev. Letters 24 (1970) 181.
[2] T. T. Wu and C. N. Yang, Phys. Rev. 137B (1965) 708; H. Abarbanel, S. Drell and F. Gilman, Phys. Rev. Letters 20 (1968) 280.
[3] S. D. Drell, D. Levy and T. M. Yan, Phys. Rev. 187 (1969) 2159; D1 (1970) 1035. See also S. Weinberg, Phys. Rev. 150 (1966) 1313.
[4] S. D. Drell and T. D. Lee, SLAC-PUB-997 (1972).
[5] K. Johnson, SLAC-PUB-1034 (1972).
[6] C. M. Ankenbrandt et al., Phys. Rev. 170 (1968) 1223; J. V. Allaby et al., Phys. Letters 23 (1966) 389, 25B (1967) 156; C. W. Akerlof et al., Phys. Rev. 159 (1967) 1138. Although the points have not been plotted, our curves are consistent with the results of G. Cocconi, Phys. Rev. 138 (1965) 165.
[7] D. P. Owen et al., Phys. Rev. 181 (1969) 1794; V. Chabend et al., Phys. Letters 38B (1972) 441.
[8] V. Chabaud et al., Phys. Letters 38B (1972) 445.
[9] S. J. Brodsky, F. E. Close and J. F. Gunion, SLAC-PUB-1012 and 973.

* * * * *